

Seesaw extended MSSM and anomaly mediation without tachyonic sleptons

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ABSTRACT: Superconformal anomalies provide an elegant and economical way to understand the soft breaking parameters in SUSY models; however, implementing them leads to the several undesirable features including: tachyonic sleptons and electroweak symmetry breaking problems in both the MSSM and the NMSSM. Since these two theories also have the additional problem of massless neutrinos, we have reconsidered the AMSB problems in a class of models that extends the NMSSM to explain small neutrino masses via the seesaw mechanism. In a recent paper, we showed that for a class of minimal left-right extensions, a built-in mechanism exists which naturally solves the tachyonic slepton problem and provides new alternatives to the MSSM that also have automatic R -parity conservation. In this paper, we discuss how electroweak symmetry breaking arises in this model through an NMSSM-like low energy theory with a singlet VEV, induced by the structure of the left-right extension and of the right magnitude. We then study the phenomenological issues and find: the LSP is an Higgsino-wino mix, new phenomenology for chargino decays to the LSP, degenerate same generation sleptons and a potential for a mild squark-slepton degeneracy. We also discuss possible collider signatures and the feasibility of dark matter in this model.

KEYWORDS: Supersymmetry Breaking, Supersymmetry Phenomenology, Supersymmetric gauge theory.

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1. Introduction

One of the leading candidates for TeV scale physics is the supersymmetric extension of the SM [1] since it resolves an outstanding SM conceptual issue: the gauge hierarchy problem (or why $M_Z \ll M_{\text{Pl}}$ is stable under radiative corrections). It also leads to gauge coupling unification as well as a candidate for dark matter of the universe if two additional assumptions are made: a grand desert until $M \sim 10^{16}$ GeV for gauge unification, and exact

R -parity for dark matter. In addition it has the potential to explain the origin of spontaneous breaking of electroweak symmetry. Of course, SUSY has to be a broken symmetry to conform with observations because no superpartner particles have been observed yet. Understanding the nature and origin of this SUSY breaking is a major challenge which has commanded a great deal of attention. An attractive and elegant mechanism is to use the superconformal anomaly [2, 3] to break supersymmetry in the manner that has been dubbed AMSB. AMSB provides an ultra-violet insensitive way to determine the soft SUSY breaking parameters [4, 5] as they depend only on the TeV scale gauge Yukawa couplings of the low energy theory. Consequently, it considerably reduces the number of arbitrary parameters of the SUSY breaking sector. It also provides a heavy gravitino which has a number of cosmological advantages.

A major problem of AMSB is that when implemented in the MSSM, it leads to negative slepton mass-squares — an unacceptable scenario since it leads to the breakdown of electric charge (sometimes called the tachyonic slepton problem). Another stumbling block to realistic AMSB model building is EWSB: the explicit μ term in the MSSM gives a $B\mu$ that is too large, while extensions like the NMSSM fail to generate a μ term that is large enough. A number of attempts have been made to extend the MSSM in order to cure these problems [6–15], usually with a focus on the tachyonic slepton problem.

Since in AMSB models the SUSY breaking profile is crucially dependent on the low energy theory, an interesting question arises as to whether AMSB still has the same problems when the MSSM is extended to accommodate neutrino masses. In a recent paper [16], we pointed out that when the MSSM is minimally extended to the SUSYLR model with $B-L=2$ triplets to implement the seesaw mechanism, the low energy particle content and interaction profile changes just enough to cure the negative slepton mass square problem. A key feature responsible for this cure is the appearance of a naturally light $SU(2)_L$ triplet and a doubly-charged singlet which have leptonic Yukawa interactions. In ref. [16], we explained how SUSYLR fixes the tachyonic slepton problem of AMSB and also noted some of the gross distinguishing features of the model — such as the appearance of $B-L=2$ triplets, doubly-charged Higgs bosons, and a pair of additional heavy Higgs doublets all with masses around the mass scale of conformal SUSY breaking, F_ϕ —typically in the tens of TeVs. Since then another paper has explored the relationship of neutrinos and AMSB in the context of deformed AMSB [17].

In this paper, which should be viewed as a sequel to ref. [16], we attempt to present a complete phenomenologically acceptable model addressing questions such as EWSB, and dark matter. A summary of our results is as follows:

- We show that the model below the F_ϕ scale is the NMSSM with a singlet superpotential mass term, μ_N . This term is necessary for EWSB and can arise from the SUSYLR framework necessary for the solution to the tachyonic slepton problem.
- One implication of the similarity to the NMSSM below the TeV scale is that the magnitude of the $B\mu$ -term is of the desired magnitude.
- We present the sparticle spectrum of the model for a generic choice of the parameters

and in particular we display the lightest superparticle which can be the dark matter of the universe. We find that same generation sleptons are degenerate and that a possibility exists for degenerate sleptons and squarks.

- We find that the mass difference between the chargino and the lightest neutralino in our model is much larger than the Minimal AMSB models where a universal scalar mass corrects the tachyonic slepton mass problem.

The paper is organized as follows: in section 2, we review the basic ingredients of the SUSYLR model that is the framework of our discussion; in section 3, we show the multi-TeV scale spectrum of the model and discuss how it solves the negative slepton mass square problem of the model; in section 4, we discuss the effective theory below the F_ϕ TeV scale and show how electroweak symmetry breaking arises. In section 5, we display the sparticle spectrum and compare it with that in some other benchmark SUSY models with different SUSY breaking mechanisms. For the allowed parameter space of our model, we find a Higgsino-wino mixture to be the LSP and mention its prospects as the dark matter of the universe. We finish with a brief discussion of the ultraviolet consequences of this model in section 6 and a conclusion.

2. Minimal supersymmetric left-right model cures the problems of anomaly mediated supersymmetry breaking: a brief review

In generic AMSB models the soft SUSY breaking parameters associated with the superfield combination $\Phi_i \Phi^{j*}$ are determined by the anomalous dimensions $\gamma_j^i(g_a, Y^{\ell mn})$ and the scaling functions $\beta_g^a(g_b, Y^{ijk})$, $\beta_Y^{ijk}(g_a, Y^{\ell mn})$ of the low energy theory:

$$(m^2)_j^i = -\frac{1}{4}|F_\phi|^2 \left[\frac{1}{2} \frac{\partial \gamma_j^i}{\partial g_a} \beta_g^a + \frac{\partial \gamma_j^i}{\partial Y^{\ell mn}} \beta_Y^{\ell mn} + \text{h.c.} \right] \quad (2.1)$$

$$a^{ijk} = \beta_Y^{ijk} F_\phi \quad (2.2)$$

$$M_{\lambda_a} = \frac{\beta_g^a}{g_a} F_\phi \quad (2.3)$$

Here F_ϕ is the SUSY breaking scale in the gauge where the conformal compensator ϕ has the form

$$\phi = 1 + F_\phi \theta^2 \quad (2.4)$$

with F_ϕ as an input parameter having a value in the 10s of TeV range. The remainder of our notational conventions can be found in appendix A.

It is clear from eq. (2.1) that when this formula is applied to the MSSM, the slepton mass-squares are negative due to the positive (asymptotically non-free) $SU(2) \times U(1)_Y$ gauge couplings' β functions and the nearly zero lepton Yukawa couplings¹. As pointed out in ref. [16], this problem is cured by extending the MSSM to SUSYLR due to the following

¹While the Yukawa coupling of τ might be significant, the first and second generation leptons have negligible Yukawa couplings

Fields	$SU(3)^c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
Q	$(3, 2, 1, +\frac{1}{3})$
Q^c	$(\bar{3}, 1, 2, -\frac{1}{3})$
L	$(1, 2, 1, -1)$
L^c	$(1, 1, 2, +1)$
Φ_a	$(1, 2, 2, 0)$
Δ	$(1, 3, 1, +2)$
$\bar{\Delta}$	$(1, 3, 1, -2)$
Δ^c	$(1, 1, 3, -2)$
$\bar{\Delta}^c$	$(1, 1, 3, +2)$
S, N	$(1, 1, 1, 0)$

Table 1: Assignment of the matter and Higgs fields' representations of the left-right symmetry group (except for $U(1)_{B-L}$ where the charge under that group is given.)

property: the effective theory below the seesaw scale v_R contains a set of $SU(2)_L$ triplets and doubly-charged fields, both having Yukawa couplings to the left- and right-handed leptons respectively. Their masses are naturally in the multi-TeV range despite the high seesaw scale due to an accidental global symmetry of the theory [18, 19]. Furthermore, provided these new couplings are of order 1, the slepton masses squares can be made positive. Thus, SUSYLR not only explains the small neutrino masses by means of the seesaw mechanism, but its marriage with AMSB cures the negative slepton mass-square problem. The resulting theory combines the predictive power of AMSB, explains neutrino masses, and retains a natural dark matter candidate due to the theory's automatic conservation of R -Parity below the right-handed scale. It also contains a mechanism for generating an appropriate singlet VEV in the effective low energy NMSSM-like superpotential. In the following subsections, we fill in the details.

2.1 The left-right model

The particle content of a SUSYLR model is shown in table 1. As the model is left-right symmetric, it contains both left- and right-handed higgs bosons — in this case $B - L = \pm 2$ triplets so that R -parity may be preserved (a task for which $B - L = 1$ doublets are not suitable). The presence of both $SU(2)_L$ and $SU(2)_R$ triplets means that parity is a good symmetry until $SU(2)_R$ breaks. While the seesaw mechanism may be achieved with only $SU(2)_R$ higgs fields, demanding parity forces the presence of left-handed triplets. The inclusion of both these fields then leads to positive left- and right-handed slepton masses.

To be explicit, the fields of table 1 transform under parity as

$$\begin{aligned}
 Q &\leftrightarrow -i\tau_2 Q^{c*} & L &\leftrightarrow -i\tau_2 L^{c*} & \Phi_a &\rightarrow \Phi_a^\dagger \\
 \Delta &\leftrightarrow \Delta^{c\dagger} & \bar{\Delta} &\leftrightarrow \bar{\Delta}^{c\dagger} & S, N &\rightarrow S^*, N^*
 \end{aligned}$$

so that the fully parity symmetric superpotential is

$$W_{\text{SUSYLR}} = W_Y + W_H + W_{\text{GSPNR}} + W_{\text{GSVNR}} \tag{2.5}$$

with

$$W_Y = \mathbf{i}y_Q^a Q^T \tau_2 \Phi_a Q^c + \mathbf{i}y_L^a L^T \tau_2 \Phi_a L^c + \mathbf{i}f_c L^{cT} \tau_2 \Delta^c L^c + \mathbf{i}f L^T \tau_2 \Delta L \quad (2.6)$$

$$W_H = (M_\Delta \phi - \lambda_S S) [\text{Tr}(\Delta^c \bar{\Delta}^c) + \text{Tr}(\Delta \bar{\Delta})] + M_S^2 \phi^2 S + \frac{1}{2} \mu_S \phi S^2 + \frac{1}{3} \kappa_S S^3 \\ + \lambda_N^{ab} N \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{3} \kappa_N N^3 \quad (2.7)$$

$$W_{\text{GSPNR}} = \frac{\lambda_A}{M_X \phi} \text{Tr}^2(\Delta \bar{\Delta}) + \frac{\lambda_A^c}{M_X \phi} \text{Tr}^2(\Delta^c \bar{\Delta}^c) \\ + \frac{\lambda_B}{M_X \phi} \text{Tr}(\Delta \Delta) \text{Tr}(\bar{\Delta} \bar{\Delta}) + \frac{\lambda_B^c}{M_X \phi} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c) \\ + \frac{\lambda_C}{M_X \phi} \text{Tr}(\Delta \bar{\Delta}) \text{Tr}(\Delta^c \bar{\Delta}^c) \\ + \frac{\lambda_S}{M_X \phi} \text{Tr}(\Delta \bar{\Delta}) S^2 + \frac{\lambda_S^c}{M_X \phi} \text{Tr}(\Delta^c \bar{\Delta}^c) S^2 + \dots \quad (2.8)$$

$$W_{\text{GSVNR}} = \frac{\lambda_D}{M_{\text{Pl}} \phi} \text{Tr}(\Delta \Delta) \text{Tr}(\Delta^c \Delta^c) + \frac{\bar{\lambda}_D}{M_{\text{Pl}} \phi} \text{Tr}(\bar{\Delta} \bar{\Delta}) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c) \\ + \frac{(\lambda_\sigma)^{ab}}{M_{\text{Pl}} \phi} \text{Tr}(\Delta \bar{\Delta}) \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{(\lambda_\sigma^c)^{ab}}{M_{\text{Pl}} \phi} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) \\ + \frac{2\lambda_\alpha \epsilon^{ab}}{M_{\text{Pl}} \phi} \text{Tr}(\Delta \Phi_a \tau_2 \Phi_b^T \tau_2 \bar{\Delta}) + \frac{2\lambda_\alpha^c \epsilon^{ab}}{M_{\text{Pl}} \phi} \text{Tr}(\Delta^c \tau_2 \Phi_a^T \tau_2 \Phi_b \bar{\Delta}^c) \\ + \frac{\lambda_N}{M_{\text{Pl}} \phi} \text{Tr}(\Delta \bar{\Delta}) N^2 + \frac{\lambda_N^c}{M_{\text{Pl}} \phi} \text{Tr}(\Delta^c \bar{\Delta}^c) N^2 \\ + \frac{\lambda_S}{M_{\text{Pl}} \phi} \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) S^2 + \frac{\lambda_M}{M_{\text{Pl}} \phi} S^2 N^2 + \dots \quad (2.9)$$

Furthermore, parity demands that the couplings be related as

$$\begin{array}{llll} y_Q^a = (y_Q^a)^\dagger & y_L^a = (y_L^a)^\dagger & f = f_c^* & M_\Delta = M_\Delta^* \\ \lambda_S = \lambda_S^* & M_S^2 = (M_S^2)^* & \mu_S = \mu_S^* & \kappa_S = \kappa_S^* \\ & \lambda_N = \lambda_N^\dagger & \kappa_N = \kappa_N^* & \end{array}$$

We have also imposed a discrete \mathbb{Z}_3 symmetry on eq. (2.5) with

$$(Q, Q^c, L, L^c, \Delta, \Delta^c, \Phi_a, N) \rightarrow \mathbf{e}^{2i\pi/3} (Q, Q^c, L, L^c, \Delta, \Delta^c, \Phi_a, N), \\ (\bar{\Delta}, \bar{\Delta}^c) \rightarrow \mathbf{e}^{4i\pi/3} (\bar{\Delta}, \bar{\Delta}^c) \quad (2.10)$$

and S invariant. This symmetry is necessary to keep one singlet light below the right-handed scale since it forbids terms such as

$$W_{\mathbb{Z}_3} = \kappa_{12} S N^2 + \kappa_{21} S^2 N + \lambda_N^c N \text{Tr}(\Delta^c \bar{\Delta}^c) \quad (2.11)$$

which would generate a large, $\mathcal{O}(v_R)$, mass for N . Yet because it is a global symmetry, it will be violated by gravitational effects² leading to eq. (2.5) containing the non-renormalizable terms of eq. (2.9) (which are accordingly suppressed by the planck scale M_{Pl}).

²For example, if a particle charged under this symmetry falls into a blackhole, there is no way to ascertain the amount of this charge the blackhole contains. This can be contrasted with a gauged symmetry where Gauss's law may be utilized to determine the charge enclosed

The superpotential eq. (2.5) must also contain the additional non-renormalizable terms given by eq. (2.8) if the theory is to preserve R -parity and be phenomenologically viable [18, 19]. These terms preserve the \mathbb{Z}_3 symmetry and are therefore suppressed by the next new scale of physics, which we have chosen to call M_X . We will show that it is possible to fix M_X in section 3, where we consider the F_ϕ scale theory.

Meanwhile, the Higgs potential given by eq. (2.7) dictates that the VEV for the right-handed superfields are

$$\langle S \rangle = \frac{M_\Delta}{\lambda_S} \phi \tag{2.12}$$

$$\langle \Delta^c \rangle \langle \bar{\Delta}^c \rangle = \langle S \rangle \left(\frac{M_\Delta \kappa_S}{\lambda_S^2} + \frac{\mu_S}{\lambda_S} \right) \phi + \frac{M_S^2}{\lambda_S} \phi^2 \tag{2.13}$$

With $M_\Delta \sim \mu_S \sim v_R \sim 10^{11}$ GeV, where v_R is the right-handed breaking scale. eq. (2.12) should be evident from the form of the superpotential; eq. (2.13) requires eq. (2.7) to be recast as

$$W_H \supset \left[-\lambda_S \text{Tr}(\Delta^c \bar{\Delta}^c) + M_S^2 \phi^2 + \frac{1}{2} \mu_S \phi S + \frac{1}{3} \kappa_S S^2 \right] S \tag{2.14}$$

The non-renormalizable terms will shift the right-handed scale VEVs by at most $\sim M_\Delta^2/M_X \ll M_\Delta$ so they may be safely be ignored. The theory then remains UV insensitive below v_R [9] and hence respects the AMSB trajectory below this scale. Yet even though the particles remain on their AMSB trajectory, the negative slepton mass-squares problem is still solved due to the additional low-scale yukawa couplings f and f_c .

To see why these yukawas survive, consider the Higgs sector of eq. (2.5) before $SU(2)_R$ breaks and setting the non-renormalizable terms to zero — essentially leaving just the terms in eq. (2.7). This superpotential has a complexified $U(6)$ symmetry³ involving the Δ 's and the Δ^c 's (similar symmetry arguments are discussed in [20], but because the authors used a parity odd singlet, there was only a complexified $U(3)$ symmetry). When $SU(2)_R$ breaks, the $U(6)$ is reduced to a $U(5)$ yielding 22 real degrees of freedom that are massless. The D -terms and the gauge fields consume 6 of these, leaving a total of 16 massless modes. The surviving 16 massless real degrees of freedom are the two doubly-charged $SU(2)_L$ singlets and the two left-handed triplets.

Only the non-renormalizable terms of eqs. (2.8) and (2.9) break the $U(6)$ symmetry, and therefore the mass of the Higgsino must be

$$\mu_{\Delta, \bar{\Delta}} \sim \mu_{DC} \sim \frac{v_R^2}{M_X} \tag{2.15}$$

The SUSY breaking bilinear terms generated by AMSB will force these masses to be at least F_ϕ giving

$$M_X \lesssim \frac{v_R^2}{F_\phi}. \tag{2.16}$$

³A complexified $U(6)$ is a $U(6)$ with its parameters taken to be complex. Its existence in eq. (2.7) can be seen by defining two new fields $\Delta \equiv (\Delta, \Delta^c)$ and $\bar{\Delta} \equiv (\bar{\Delta}, \bar{\Delta}^c)$ —which are complex 6-vectors — and combining the trace over each separately to $\text{Tr}(\Delta \bar{\Delta})$

Thus, the scale of new physics is determined by the right-handed scale and the SUSY breaking scale.

The mass matrix for the left-handed triplets and doubly-charged Higgses have a similar form, here we state the doubly-charged matrix:

$$\mathcal{M}_{DC} = \mu_{DC}^2 \begin{pmatrix} 1 & 1 - \epsilon_{\Delta} \\ 1 - \epsilon_{\Delta} & 1 \end{pmatrix} \quad (2.17)$$

where $\mu_{DC} \simeq F_{\phi}$ and $\epsilon_{\Delta} = 1 - \frac{B_{\Delta}}{\mu_{DC}}$. The eigenvalues of this mass matrix are $m_{DC}^2 = \epsilon_{\Delta} \mu_{DC}^2$ and $M_{DC}^2 = 2\mu_{DC}^2$. Since ϵ_{Δ} depends on μ_{DC} , and μ_{DC} can be adjusted through the coupling it contains, one doubly-charged Higgs can be made light. On the whole, we expect the two doubly charged scalar masses to be above 1 TeV (for the lighter one) and F_{ϕ} (for the heavier one). Note that there is no such splitting between the fermionic partners, which remain heavy with a mass of about μ_{DC} . A similar argument applies to the left-handed triplets.

Finally, because the masses of the $SU(2)_L$ triplets and the doubly-charged particles will be around F_{ϕ} , they are of the correct size to influence the low-scale theory: if the masses had been large, $F_{\phi} \ll \mu_{DC} \ll v_R$, then they would have merely introduced another trajectory preserving threshold that decoupled from the low scale theory. However, because these particles remain in the low-scale theory, the effect of their couplings is important. For the sleptons the relevant terms are

$$W \supset f_c \Delta^{c--} e^c e^c + \mathbf{i} f L^T \tau_2 \Delta L \quad (2.18)$$

with the surviving yukawa couplings f_c and f providing positive mass-squares to the scalar leptons⁴

To make this explicit we write down the slepton masses with the contributions of these additional interactions (taking the $SU(2)_L \times U(1)_Y$ gauge couplings to be g_2 and g_1 respectively):

$$\begin{aligned} m_{ec}^2 = & \frac{1}{2} \frac{|F_{\phi}|^2}{(16\pi^2)^2} \left[8f_c^{\dagger} (Y_L^a)^T (Y_L^a)^* f_c + 12(Y_L^a)^{\dagger} f f^{\dagger} Y_L^a \right. \\ & + 8f_c^{\dagger} f_c \left[(Y_L^a)^{\dagger} Y_L^a + 4f_c^{\dagger} f_c + \text{Tr}(f_c^{\dagger} f_c) \right] + 4(Y_L^a)^{\dagger} Y_L^a \left[(Y_L^b)^{\dagger} Y_L^b + 2f_c^{\dagger} f_c \right] \\ & + 2(Y_L^a)^{\dagger} Y_L^b \left[2(Y_L^b)^{\dagger} Y_L^a + \text{Tr}\left(3(Y_Q^b)^{\dagger} Y_Q^a + (Y_L^b)^{\dagger} Y_L^a\right) + 4(\lambda_N^{cb})^* \lambda_N^{ca} \right] \\ & \left. - 2g_1^2 \left(24f_c^{\dagger} f_c + 3(Y_L^a)^{\dagger} Y_L^a + 26g_1^2 \right) - 6g_2^2 (Y_L^a)^{\dagger} Y_L^a + \text{h.c.} \right] \quad (2.19) \end{aligned}$$

$$m_L^2 = \frac{1}{2} \frac{|F_{\phi}|^2}{(16\pi^2)^2} \left[6f(Y_L^a)^T (Y_L^a)^* f^{\dagger} + 4Y_L^a f_c^{\dagger} f_c (Y_L^a)^{\dagger} \right]$$

⁴Note that slepton mass squares can also be positive for theories with a right handed scale lower than 10^{11} GeV. We choose the high scale version since neutrino masses in this case do not require any fine tuning of Yukawa couplings.

$$\begin{aligned}
 & + 6 \left[(Y_L^a)^\dagger Y_L^a + 12 f f^\dagger + 2 \text{Tr} (f^\dagger f) \right] f f^\dagger + 2 \left[Y_L^b (Y_L^b)^\dagger + 3 f f^\dagger \right] Y_L^a (Y_L^a)^\dagger \\
 & + Y_L^b (Y_L^a)^\dagger \left[2 Y_L^a (Y_L^b)^\dagger + \text{Tr} \left(3 (Y_Q^b)^\dagger Y_Q^a + (Y_L^b)^\dagger Y_L^a \right) + 4 (\lambda_N^{cb})^* \lambda_N^{ca} \right] \\
 & - g_1^2 \left(18 f f^\dagger + 3 Y_L^a (Y_L^a)^\dagger + 13 g_1^2 \right) - 3 g_2^2 \left(14 f f^\dagger + Y_L^a (Y_L^a)^\dagger + 3 g_2^2 \right) + \text{h.c.} \Big] \quad (2.20)
 \end{aligned}$$

Taking

$$m_{\text{an}} = \frac{F_\phi}{16\pi^2}, \quad (2.21)$$

assuming that f , f_c are diagonal in flavor space (an assumption required to satisfy constraints from lepton flavor violating experiments [21]), and neglecting the first and second generation yukawa couplings simplifies eqs. (2.19) and (2.20) to

$$m_{e^c}^2 = m_{\text{an}}^2 [40 f_{c1}^4 + 8 f_{c1}^2 (f_{c2}^2 + f_{c3}^2) - 48 f_{c1}^2 g_1^2 - 52 g_1^4] \quad (2.22)$$

$$m_e^2 = m_{\text{an}}^2 [84 f_1^4 + 12 f_1^2 (f_2^2 + f_3^2) - 6 f_1^2 (3 g_1^2 + 7 g_2^2) - 13 g_1^4 - 9 g_2^4] \quad (2.23)$$

for the first generation.⁵ We then only need

$$f_1(F_\phi) \simeq f_2(F_\phi) \simeq f_{c1}(F_\phi) \simeq f_{c2}(F_\phi) \gtrsim 0.6 \quad (2.24)$$

to make the sleptons positive (from the detailed analysis of section 5.2).

These couplings and the masses of the doubly-charged field and the left-handed triplets are experimentally constrained from muonium-antimuonium oscillations [22] which demands that

$$\frac{f_{c1} f_{c2}}{4\sqrt{2} m_{DC}^2} \approx \frac{f_1 f_2}{4\sqrt{2} m_{\Delta, \bar{\Delta}}^2} < 3 \times 10^{-3} G_F; \quad (2.25)$$

The minimum f values that satisfies eq. (2.24) implies a lower bound on the masses of the doubly-charged and left-handed triplet Higgs field to be about $m_{DC}, m_\Delta \geq 2$ TeV. The lighter end of this range is clearly accessible at the LHC .

It is worth noting that even though the f, f_c are diagonal, one may obtain large neutrino mixing. As already noted, the neutrino masses arise from the type I seesaw [23–27] formula given by:

$$\begin{aligned}
 \mathcal{M}_\nu & = -M_D^T M_R^{-1} M_D \\
 & = \frac{v_{wk}^2 \sin^2 \beta}{v_R^2} y_\nu^T f_c^{-1} y_\nu \quad (2.26)
 \end{aligned}$$

Note that the Yukawa coupling matrix y_ν is arbitrary and can be easily arrange to give large mixings even though f is diagonal and we can fit the neutrino data by appropriate choice of parameters.

⁵The expressions for the smuon may be gotten by taking $f_1 \leftrightarrow f_2$ and $f_{c1} \leftrightarrow f_{c2}$.

3. Between Scales: v_R to F_ϕ

Once $SU(2)_R$ breaks around the seesaw scale of 10^{11} GeV, the effective theory contains the NMSSM, an extra set of higgs doublets, a pair of left-handed triplets, and the doubly-charged fields⁶. The non-renormalizable terms of eq. (2.5) also influence the form of the lower scale theory and produce some important effects that aid in construction of a realistic low-energy theory. One significant contribution comes from the higher dimensional operators: the generation of a mass term for N . Specifically the terms

$$\frac{\lambda_N^c}{M_{\text{Pl}}\phi} \text{Tr}(\Delta^c \bar{\Delta}^c) N^2 + \frac{\lambda_M}{M_{\text{Pl}}\phi} S^2 N^2 \quad (3.1)$$

generate a superpotential term of $\mu_N \phi N^2$ when Δ^c , $\bar{\Delta}^c$, and S get a VEV. The mass μ_N is given by⁷

$$\mu_N \equiv \frac{\lambda_N^c}{M_{\text{Pl}}} \langle \underline{\Delta}^c \rangle \langle \underline{\bar{\Delta}}^c \rangle + \frac{\lambda_M}{M_{\text{Pl}}} \langle \underline{S} \rangle^2 \simeq \frac{v_R^2}{M_{\text{Pl}}} \quad (3.2)$$

Because the v_R threshold preserves the AMSB trajectory, this explicit mass term produces a SUSY breaking bilinear term proportional to F_ϕ

$$\int d^2\theta \mu_N \phi N^2 \supset \mu_N F_\phi N^2 \equiv b_N N^2 \quad (3.3)$$

with b_N given as

$$b_N = \mu_N F_\phi \simeq \frac{v_R^2}{M_{\text{Pl}}} F_\phi. \quad (3.4)$$

In section 4 this term will be shown to play an important role in EWSB; for now it suffices to note that if b_N is to be of the expected order of M_{SUSY}^2 , then the right-handed scale must be around $v_R \simeq 10^{11}$ GeV. Constraining v_R automatically determines the scale of new physics M_X from eq. (2.16): $M_X \lesssim 10^{16} - 10^{18}$ GeV. The end result is that the order of magnitude of all the scales of the theory are fixed.

Furthermore, the non-renormalizable terms can also be used to simplify the low-energy theory, though this is not necessary. Consider the terms

$$\frac{(\lambda_\sigma^c)^{ab}}{M_{\text{Pl}}\phi} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi_a \tau_2 \Phi_b^T \tau_2) + \frac{2\lambda_\alpha^c \epsilon^{ab}}{M_{\text{Pl}}\phi} \text{Tr}(\Delta^c \tau_2 \Phi_a^T \tau_2 \Phi_b \bar{\Delta}^c) \quad (3.5)$$

which yield a low energy mass matrix for the Φ 's that is not symmetric between Φ_1 and Φ_2 (due to the second term). The asymmetry generates an operator of the form:

$$W \supset \mathbf{i} M H_{u2} \tau_2 H_{d1} \quad (3.6)$$

without the corresponding $H_{u1} H_{d2}$ term. This allows a large mass, say of order F_ϕ , for H_{u2} and H_{d1} while leaving H_{u1} and H_{d2} light. The resulting VEVs for $\underline{H_{u2}}$ and $\underline{H_{d1}}$ will then be suppressed by M and will not play a role in the theory below F_ϕ .

⁶The resulting theory with the additional particle content might be aptly labeled the NMSSM++

⁷We choose to denote the scalar component of the superfield X as \underline{X} to avoid confusion between the superfield and its scalar component. This allows us to write more meaningful expressions such as $\langle X \rangle / \langle \underline{X} \rangle = \phi$

Finally, as discussed in section 2, the non-renormalizable terms yield masses around F_ϕ for the left-handed triplets as well as the doubly-charged fields. These fields therefore decouple from the electroweak scale theory along with the extra bi-doublet due to the doublet-doublet splitting mechanism discussed above. This leaves the low energy theory as the NMSSM and we use this to explore electroweak symmetry breaking as well as the remaining consequences of the low-energy theory.

4. EWSB

Naively it would be expected that the resulting low-energy theory is merely the NMSSM (since the remaining particle content is precisely that theory), but if this were the case, the model would not be able to achieve a realistic mass spectrum — the singlet N would get a very small VEV, and the Higgsino would be lighter than allowed by experiment [28]. The origin of this problem is best illustrated with a toy model:

4.1 Toy exposition

Consider a superpotential given by

$$W_{\text{toy}} = \frac{1}{3}\kappa N^3 \tag{4.1}$$

where N is a singlet field with no gauge symmetries. The resulting scalar potential, including SUSY breaking, is

$$V_{\text{toy}} = \kappa^2 |N|^4 + \frac{1}{3}(a_\kappa N^3 + a_\kappa^* N^{*3}) + m_N^2 |N|^2. \tag{4.2}$$

Assuming the parameters κ , a_κ , and $\langle N \rangle$ are real, the minimization condition for eq. (4.2) is

$$2\kappa^2 \langle N \rangle^2 + a_\kappa \langle N \rangle + m_N^2 = 0 \tag{4.3}$$

and the solution is given as

$$\langle N \rangle = \frac{-a_\kappa \pm \sqrt{a_\kappa^2 - 8\kappa^2 m_N^2}}{2\kappa^2} \tag{4.4}$$

The soft couplings a_κ and m_N are determined by AMSB via eqs. (2.1) and (2.2):

$$\begin{aligned} a_\kappa &= \frac{F_\phi}{16\pi^2} 6\kappa^3 \\ m_N^2 &= \frac{|F_\phi|^2}{(16\pi^2)^2} 12\kappa^4 \end{aligned} \tag{4.5}$$

Substituting these into eq. (4.4) yields

$$\langle N \rangle = \frac{F_\phi}{16\pi^2} \frac{\kappa}{4} (-6 \pm \sqrt{-60}) \tag{4.6}$$

and the large negative under the radical demonstrates the inability to achieve a real, non-zero VEV in this model.

The source of the problem can be identified by examining the potential of N . To expose the difficulty, it is helpful to define

$$x \equiv \frac{\kappa \langle N \rangle}{m_{\text{an}}} \tag{4.7}$$

and re-write eq. (4.2) as

$$\frac{\langle V_{\text{toy}} \rangle}{4m_{\text{an}}^4} = \frac{1}{4\kappa^2} x^4 + x^3 + 3\kappa^2 x^2 \tag{4.8}$$

where the AMSB expressions of eq. (4.5) have been substituted. For the potential to have a non-trivial minimum, it is necessary that the cubic term dominate for some value of x (since this term is the only one that provides a negative contribution to the potential); however, for large κ , the x^2 term will always be larger than the cubic term. Meanwhile, for small κ the quartic term will dominate the expression. Therefore, if there is any chance for the x^3 term to create a minimum other than zero, it must be that $\kappa \simeq 1$. This leaves the potential as

$$\frac{\langle V_{\text{toy}} \rangle}{4m_{\text{an}}^4} = \frac{1}{4} x^4 + x^3 + 3x^2 \tag{4.9}$$

where it now becomes clear that neither large x , $x \sim 1$, nor small x will have the cubic term dominate the expression — leaving the only minimum as the trivial one. Thus, the heart of the problem is that AMSB predicts the cubic term's coefficient such that it will always be weaker than either the quartic or quadratic regardless of the parameter regime.

The same problem carries over to the full NMSSM, as pointed out in [28]. In this model, the additional coupling of N to H_u and H_d does not alter the relative strengths of N 's quartic, cubic, or quadratic terms, but it does add a linear term to the potential, $a_\lambda v_u v_d N$. The induced linear term shifts the trivial minimum away from zero, but keeps it small. The minimization condition for N can then be approximated as

$$\tilde{\mu}_N^2 \langle N \rangle - \frac{1}{2\sqrt{2}} a_\lambda v^2 \sin 2\beta = 0 \tag{4.10}$$

with $\tilde{\mu}_N^2 \simeq m_{\text{an}}^2$ being essentially the AMSB predicted soft SUSY breaking mass for N . The maximum value occurs when $\sin 2\beta = 1$ so we have that

$$\langle N \rangle \lesssim \frac{a_\lambda v^2}{2\tilde{\mu}_N^2 \sqrt{2}} \simeq \frac{1}{2\sqrt{2}} \frac{v^2}{m_{\text{an}}} \simeq 22 \text{ GeV} \tag{4.11}$$

The small $\langle N \rangle$ then results in a chargino mass which falls below the LEP II bound of about 94 GeV.

Given this limitation of the NMSSM, it is desirable to explore methods that either alter the relative strengths of the terms or yield a large tadpole term for N . The former may be done by adding vector-like matter (as in [6]), while the latter was explored in [28] by introducing a linear term for N . We propose here a different solution that alters the relative strengths and is already present in the model.

4.2 Low energy theory

The superpotential of eq. (2.5) contains in its non-renormalizable terms the key to solving the small $\langle N \rangle$ problem: as discussed in section 3, the terms of eq. (2.9) generate a mass term for N given by eq. (3.2). This mass term then yields a SUSY breaking bilinear term given by eq. (3.4). The size of b_N is quite conveniently around the SUSY breaking scale and also provides a means of turning the net mass-square of N negative. To establish this property we now turn to the effective M_{SUSY} -scale theory.

The effective superpotential responsible for EWSB (valid for $M_{\text{SUSY}} < Q \ll F_\phi$) is

$$\begin{aligned}
 W|_{M_{\text{SUSY}}} = & \mathbf{i}y_u Q^T \tau_2 H_u u^c + \mathbf{i}y_d Q^T \tau_2 H_d d^c + \mathbf{i}y_e L^T \tau_2 H_d e^c \\
 & + \mathbf{i}\lambda N H_u^T \tau_2 H_d + \frac{1}{2}\mu_N N^2 + \frac{1}{3}\kappa N^3
 \end{aligned} \tag{4.12}$$

and the SUSY breaking potential is

$$\begin{aligned}
 V_{\text{SB}}|_{M_{\text{SUSY}}} = & m_Q^2 Q^\dagger Q + m_{u^c}^2 u^{c\dagger} u^c + m_{d^c}^2 d^{c\dagger} d^c + m_L^2 L^\dagger L + m_{e^c}^2 e^{c\dagger} e^c \\
 & + m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + m_N^2 N^* N \\
 & + [\mathbf{i}a_u Q^T \tau_2 H_u u^c + \mathbf{i}a_d Q^T \tau_2 H_d d^c + \mathbf{i}a_e L^T \tau_2 H_d e^c + \text{h.c.}] \\
 & + \left[\mathbf{i}a_\lambda N H_u^T \tau_2 H_d - \frac{1}{2}b_N N^2 + \frac{1}{3}a_\kappa N^3 + \text{h.c.} \right] \\
 & - \frac{1}{2}(M_3 \lambda_3 \lambda_3 + M_2 \lambda_2 \lambda_2 + M_1 \lambda_1 \lambda_1 + \text{h.c.})
 \end{aligned} \tag{4.13}$$

The resulting Higgs sector potential is

$$V = V_F + V_D + V_{\text{SB}} \tag{4.14}$$

with V_F and V_D the typical SUSY contribution:

$$V_F = |\lambda|^2 |N|^2 \left(|H_u|^2 + |H_d|^2 \right) + |\mathbf{i}\lambda H_u^T \tau_2 H_d + \mu_N N + \kappa N^2|^2 \tag{4.15}$$

$$V_D = \frac{1}{8}(g_1^2 + g_2^2) \left(|H_u|^2 - |H_d|^2 \right)^2 + \frac{1}{2}g_2^2 |H_u^\dagger H_d|^2 \tag{4.16}$$

The potential of eq. (4.14) can be made to spontaneously break electroweak symmetry giving

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} \quad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} \quad \langle N \rangle = \frac{n}{\sqrt{2}} \tag{4.17}$$

and we take the usual definitions: $v_u = v \sin \beta$ and $v_d = v \cos \beta$. The minimization conditions are

$$m_{H_u}^2 - \frac{1}{8}(g_2^2 + g_1^2) v^2 \cos 2\beta + \frac{1}{2}\lambda^2 (n^2 + v^2 \cos^2 \beta) - \frac{n}{\sqrt{2}} \left(\tilde{a}_\lambda + \frac{\lambda \kappa n}{\sqrt{2}} \right) \cot \beta = 0 \tag{4.18}$$

$$m_{H_d}^2 + \frac{1}{8}(g_2^2 + g_1^2) v^2 \cos 2\beta + \frac{1}{2}\lambda^2 (n^2 + v^2 \sin^2 \beta) - \frac{n}{\sqrt{2}} \left(\tilde{a}_\lambda + \frac{\lambda \kappa n}{\sqrt{2}} \right) \tan \beta = 0 \tag{4.19}$$

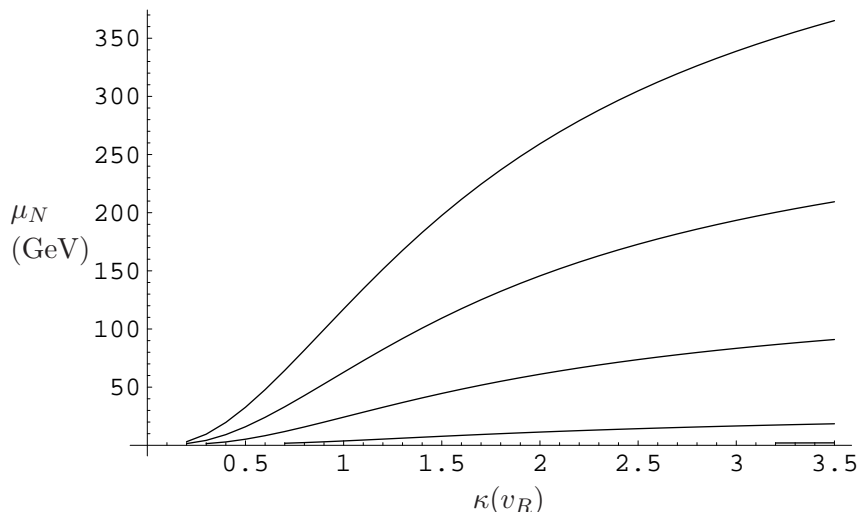


Figure 1: Constant n contours in the μ_N - $\kappa(v_R)$ plane where the curves, from top to bottom, correspond to $n = -10000, -7500, -5000, -2500$ and -1000 GeV. A constant value of $\tan\beta = 3.25$ has been assumed with $F_\phi = 33$ TeV and $\lambda(v_R) = 0.5$.

$$\tilde{m}_N^2 + \kappa^2 n^2 + \frac{1}{2}\lambda^2 v^2 + \frac{n\tilde{a}_\kappa}{\sqrt{2}} - \frac{1}{2}v^2\left(\frac{\tilde{a}_\lambda}{n\sqrt{2}} + \lambda\kappa\right)\sin 2\beta = 0 \quad (4.20)$$

The tilded variables are introduced to display the deviations from the usual NMSSM due to the presence of the term μ_N in eq. (4.12). These constructs are defined as

$$\tilde{a}_\lambda \equiv a_\lambda + \lambda\mu_N \quad (4.21)$$

$$\tilde{a}_\kappa \equiv a_\kappa + 3\kappa\mu_N \quad (4.22)$$

$$\tilde{m}_N^2 \equiv m_N^2 + \mu_N^2 - b_N \quad (4.23)$$

Of particular interest is eq. (4.23), which may be recast using eq. (3.4) of section 3:

$$\begin{aligned} \tilde{m}_N^2 &= m_N^2 + \mu_N^2 - \mu_N F_\phi \\ &\approx m_N^2 - \mu_N F_\phi \\ &\simeq \left(\frac{\lambda^4}{(16\pi^2)^2} F_\phi - \mu_N\right) F_\phi \end{aligned}$$

The second line follows from the fact that $\mu_N \sim \mathcal{O}\left(\frac{M_{\text{SUSY}}^2}{F_\phi}\right) \sim \mathcal{O}\left(\frac{F_\phi}{(16\pi^2)^2}\right)$ and therefore the μ_N^2 term is negligible compared to the other terms. The last line uses the AMSB expression for the scalar mass-squared, assuming it is dominated by the λ contribution. As can be seen, due to the λ^4 suppression, it is relatively easy to adjust μ_N to the appropriate value to make \tilde{m}_N^2 negative and therefore induce a singlet VEV of the correct size. Given that $\lambda(M_{\text{SUSY}}) \lesssim 0.5$ (from constraints of perturbativity to the right-handed scale) and that $\mu = \frac{\lambda n}{\sqrt{2}}$, it is only necessary for $n \gtrsim 300$ GeV to achieve chargino masses above the LEP II bound.

Figure 1 shows that such values are easily attainable in this situation. In the figure, constant n contours are plotted in the μ_N - $\kappa(v_R)$ plane treating the VEVs of the Higgs

doublets as constant background values with $\tan \beta = 3.25$, $F_\phi = 33$ TeV, and $\lambda(v_R) = 0.5$. The ample parameter space therefore demonstrates that this inherent property of our model easily provides a means to resolve the conflict between AMSB and the NMSSM.

The resulting mass spectrum for this $\tilde{\text{N}}\text{MSSM}$ is quite similar to the NMSSM (see [29])—particularly for the scalar and charged Higgses⁸ whose mass matrices are given by

$$M_S^2 = \begin{pmatrix} \frac{v_u^2}{4}(g_1^2 + g_2^2) + \frac{nv_d}{\sqrt{2}v_u}\tilde{A}_\Lambda & \frac{v_d v_u}{4}(4\lambda^2 + g_1^2 + g_2^2) - \frac{n}{\sqrt{2}}\tilde{A}_\Lambda & \lambda^2 n v_u - \frac{v_d}{\sqrt{2}}\tilde{a}_\lambda - \lambda \kappa v_d n \\ \frac{v_d v_u}{4}(4\lambda^2 + g_1^2 + g_2^2) - \frac{n}{\sqrt{2}}\tilde{A}_\Lambda & \frac{v_u^2}{4}(g_1^2 + g_2^2) + \frac{nv_u}{\sqrt{2}v_d}\tilde{A}_\Lambda & \lambda^2 n v_d - \frac{v_u}{\sqrt{2}}\tilde{a}_\lambda - \lambda \kappa v_u n \\ \lambda^2 n v_u - \frac{v_d}{\sqrt{2}}\tilde{a}_\lambda - \lambda \kappa v_d n & \lambda^2 n v_d - \frac{v_u}{\sqrt{2}}\tilde{a}_\lambda - \lambda \kappa v_u n & 2n^2 \kappa^2 + \frac{n}{\sqrt{2}}\tilde{a}_\kappa + \frac{v_u v_d}{\sqrt{2}n}\tilde{a}_\lambda \end{pmatrix} \quad (4.24)$$

and

$$M_C^2 = \frac{v^2}{2v_d v_u} \left[\sqrt{2}n\tilde{A}_\Lambda + v_d v_u \left(\frac{1}{2}g_2^2 - \lambda^2 \right) \right]; \quad (4.25)$$

defining $\tilde{A}_\Lambda \equiv \tilde{a}_\lambda + \frac{\lambda \kappa n}{\sqrt{2}}$.

On the other hand, the pseudoscalar mass matrix gets a contribution from the b_N term which is rather large and typically guarantees that the heavier pseudoscalar is mostly singlet. Its mass matrix is given by:

$$M_P^2 = \begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{A}_\Lambda \frac{v^2 n}{v_u v_d} & \frac{v}{\sqrt{2}}(a_\lambda - \lambda \mu_N - \sqrt{2}\lambda \kappa n) \\ \frac{v}{\sqrt{2}}(a_\lambda - \lambda \mu_N - \sqrt{2}\lambda \kappa n) & \frac{v_u v_d}{n\sqrt{2}}(\tilde{a}_\lambda + 2\lambda \kappa n \sqrt{2}) - 3\tilde{a}_\kappa \frac{n}{\sqrt{2}} + 2b_N + 8\kappa \mu_N \frac{n}{\sqrt{2}} \end{pmatrix} \quad (4.26)$$

The neutralino and chargino mass matrices remain similar to the NMSSM, and in the bases $(\tilde{B}, \tilde{W}, \tilde{H}_u, \tilde{H}_d, \tilde{N})$, $(\tilde{W}^+, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-)$ they are:

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & M_Z \sin \beta \sin \theta_W & -M_Z \cos \beta \sin \theta_W & 0 \\ 0 & M_2 & -M_Z \sin \beta \cos \theta_W & M_Z \cos \beta \cos \theta_W & 0 \\ M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & 0 & -\frac{\lambda}{\sqrt{2}}n & -\frac{\lambda}{\sqrt{2}}v_d \\ -M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \cos \theta_W & -\frac{\lambda}{\sqrt{2}}n & 0 & -\frac{\lambda}{\sqrt{2}}v_u \\ 0 & 0 & -\frac{\lambda}{\sqrt{2}}v_d & -\frac{\lambda}{\sqrt{2}}v_u & \sqrt{2}\kappa n + \mu_N \end{pmatrix} \quad (4.27)$$

$$M_{\chi^\pm} = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}; \quad X = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix} \quad (4.28)$$

respectively.

⁸Simply substitute the appropriate variables with their tilded form: $(a_\kappa, a_\lambda, m_N^2) \rightarrow (\tilde{a}_\kappa, \tilde{a}_\lambda, \tilde{m}_N^2)$. Typically, however, μ_N is rather small and so the untilded variables make a good approximation to the tilded ones.

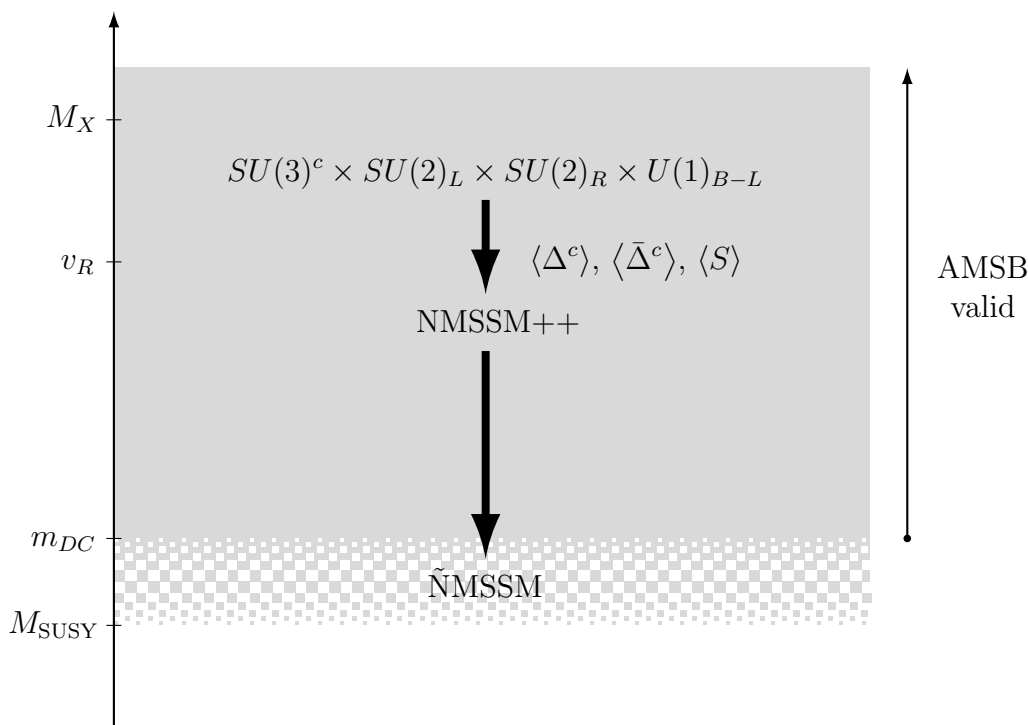


Figure 2: A schematic of the SUSYLR+AMSB model showing the complete picture through all the energy scales.

4.3 A brief summary of scales

With EWSB achieved and the mass spectrum given, we now have a complete picture of the physics starting at the high scale v_R and coming down to the electroweak scale. The theory starts as a parity-conserving SUSYLR model with AMSB generating the SUSY breaking, breaks down to the NMSSM++ (but without introducing new SUSY breaking effects), and finally ends up at M_{SUSY} as the $\tilde{\text{NMSSM}}$ (as elucidated in figure 2). We may now turn our attention to the rich phenomenological consequences of this theory.

5. Phenomenology

The following numerical values are based on our parameter running scheme. We run the gauge coupling values from the electroweak scale to the right-handed scale, $v_R = 2 \times 10^{11}$ taking the F_ϕ threshold into account by decoupling the triplets and doubly charged fields. Yukawa couplings are then inputs at the right-handed scale: the third generation values for the SM couplings (y_Q, y_L) and all three generations of the seesaw couplings (f, f_c). These are evolved down to the SUSY scale [30, 31]. Because of parity $f = f_c$ at the right-handed scale and all of the off diagonal terms are taken to be negligible due to lepton flavor violating constraints. We also assume that the first and second generation seesaw couplings are equal ($f_2 = f_1$) for simplicity. Soft terms follow their AMSB trajectory, given

Sfermions	Masses (GeV)	Bosinos and Higgses	Masses (GeV)
\tilde{u}	623	\tilde{g}	472
\tilde{d}	627	$\tilde{\chi}_1^0$	417
\tilde{e}	623/484	$\tilde{\chi}_2^0$	472
$\tilde{\nu}_e$	621/479	$\tilde{\chi}_3^0$	561
\tilde{u}^c	654	$\tilde{\chi}_4^0$	713
\tilde{d}^c	662	$\tilde{\chi}_5^0$	1644
\tilde{e}^c	587/438	$\tilde{\chi}_1^+$	421
\tilde{t}_1	496	$\tilde{\chi}_2^+$	565
\tilde{b}_1	547	h^0	116
$\tilde{\tau}_1$	587/438	A^0	518
$\tilde{\nu}_\tau$	621/479	H^0	523
\tilde{t}_2	641	H^+	526
\tilde{b}_2	603	A_2^0	2086
$\tilde{\tau}_2$	625/485	H_3^0	1284

Table 2: Mass spectrum for the LR-AMSB point given in figure 3. Slepton masses are reported for $f_1(v_R) = f_3(v_R) = 3.5/1.4$. Higgs masses are also reported here as well as the mostly singlino neutralino.

by eqs. (2.1), (2.2) and (2.3) down to the F_ϕ scale, below which the soft terms are evolved to the SUSY scale using the usual RGE of the NMSSM [32]. Note that due to the mass splitting between the Higgsinos and Higgses of both the doubly charged and left-handed triplets described in section 2.1, there will be some corrections to the SUSY RGEs. These corrections will depend on the mass splitting and will be fairly small.

Numerical results will be compared to popular SUSY breaking models: mSUGRA, mGMSB and Minimal AMSB — an AMSB in which the slepton mass problem is fixed by adding a universal mass, m_0 to all sfermion soft masses [3]. Note that slepton phenomenological comparisons to Minimal AMSB also apply to [11] since the additional R -parity violating lepton sector Yukawa coupling is analogous to adding a universal slepton mass.

5.1 The spectrum

Before engaging in the full details of the various sectors of the model, it is helpful to take a step back and look at the overall spectrum. figure 3 examines the bosinos and figure 4 the sfermions in this model and compares their masses to similar points in parameter space for mSUGRA, mGMSB and Minimal AMSB calculated from isajet [33] (matching between the different points were done based on the gluino mass). The columns of the bosino chart, figure 3, from left to right are gluino, neutralino and chargino. The columns of the sfermion chart, figure 4 from left to right are: left-handed first generation, right-handed first generation, lightest mass third generation (and third generation neutrinos), heaviest third generation and gluinos — for comparison with the bosino chart. The Higgses and the mostly singlino neutralino have not been included to keep from cluttering the plots, although their masses are reported in table 2.

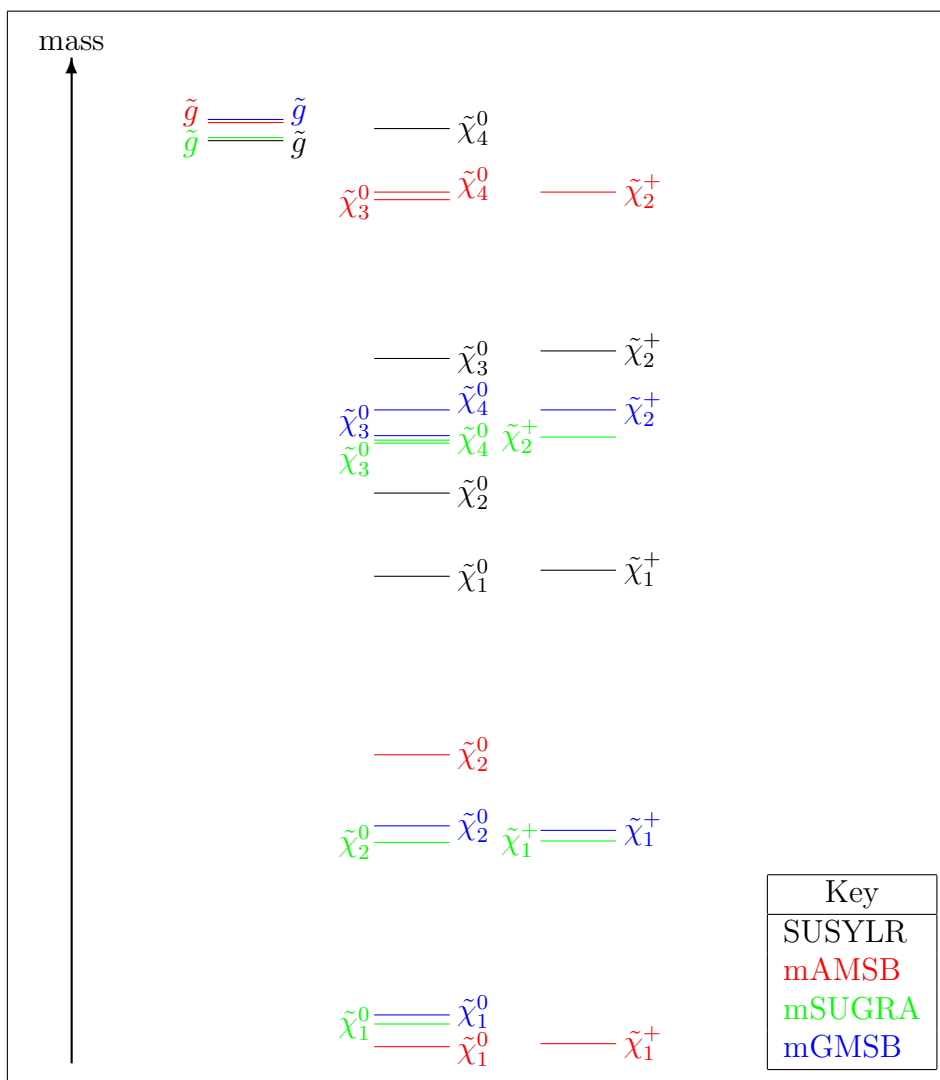


Figure 3: From left to right, columns correspond to charginos, neutralinos and gluino masses at $\tan\beta = 3.25$ and $\text{sgn}\mu = +1$. The parameter points are: $F_\phi = 33$ TeV, $f_1(v_R) = f_3(v_R) = 3.5$ for LR-AMSB; $m_0 = 209$ GeV, $m_{\frac{1}{2}} = -300$ GeV and $A_0 = 265$ GeV for mSUGRA; $\Lambda = 99$ TeV, $M_{\text{mess}} = 792$ TeV and $N_5 = 1$ for mGMSB; $F_\phi = 33$ TeV and $m_0 = 645$ GeV for Minimal AMSB (here we also matched to the lightest slepton).

The most striking general feature of these figures is the degeneracy of the spectrum between colored and electroweak particles in LR-AMSB. While this is very dependent on the seesaw couplings (table 2 shows slepton masses that are lighter than the squarks due to smaller values of the seesaw couplings), it is a possibility that is difficult to achieve in other models. table 2 also shows the Higgs masses. Here H_3 and A_2 are the mostly singlet scalar and pseudoscalar. Due to the large size of the singlet VEV, these fields decouple from the spectrum as does the mostly singlino $\tilde{\chi}_3^0$. The neutral scalar Higgs masses stated

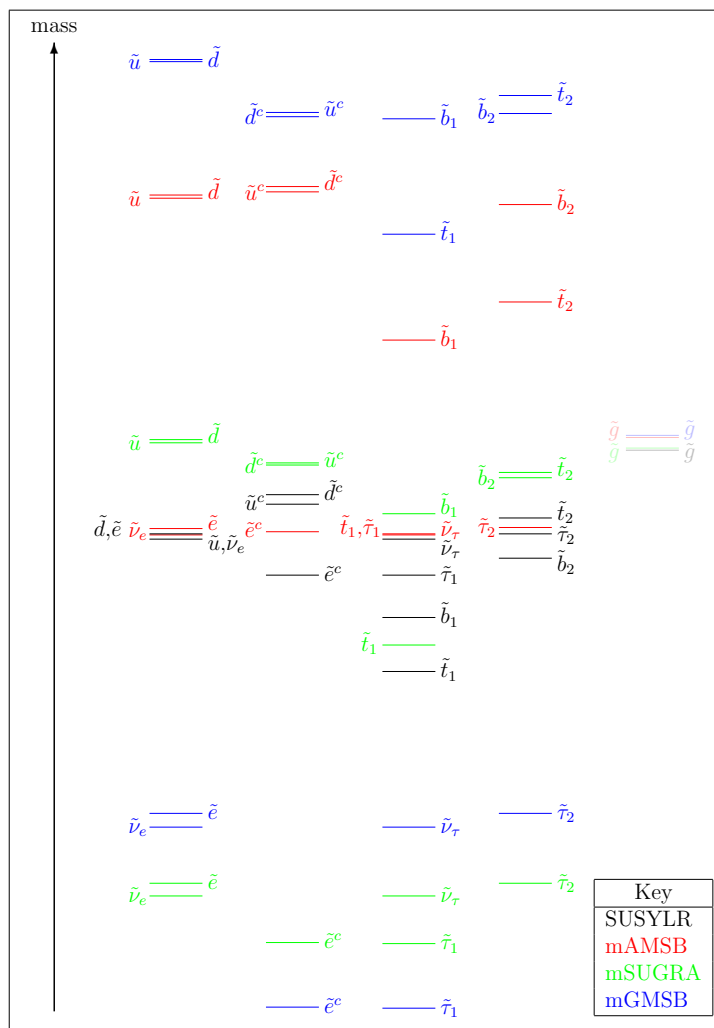


Figure 4: From left to right, columns correspond to first generation left-handed, first generation right-handed, lightest third generation and heaviest third generation sfermions. The final column consists of gluino masses for comparison with figure 3. Input parameters are as given in figure 3.

include the full radiative corrections due to top and stop loops [34]. These corrections need $m_{\tilde{t}} \gtrsim 600$ GeV which implies $F_\phi \gtrsim 33$ TeV to allow the Higgs to be above the LEP II bound. In the following subsections, we will continue to explore this spectrum, focusing on the sleptons, squarks and finally the neutralinos and charginos.

5.2 Sleptons

We start this discussion by analyzing the seesaw yukawa couplings f and f_c . In all the work that follows, we take their maximum value at v_R to be $\sim \sqrt{4\pi} \sim 3.5$ based on perturbativity arguments. Of immediate note is the fixed point-like behavior of these couplings. This can be seen in figure 5, which plots f_{c1} versus the log of the energy scale for initial values

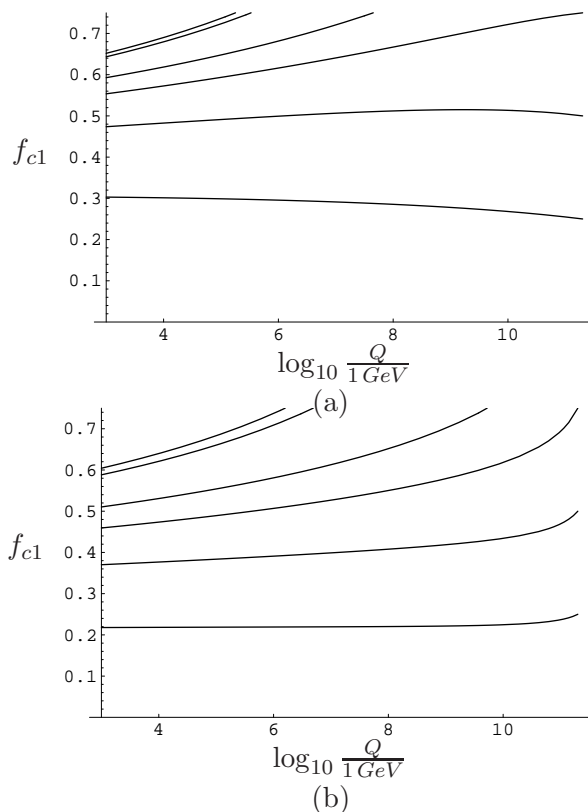


Figure 5: Plots of f_{c1} versus the log of the energy scale. The lines correspond, in ascending order, to $f_1(v_R)$ values of 0.25, 0.5, 0.75, 1, 2.25 and 3.5 for (a) $f_3(v_R) = 0$ and (b) $f_3(v_R) = 3.5$.

	f_3	f_1	f_{c3}	f_{c1}
Fixed Point Value	0.64	0.64	0.67	0.67

Table 3: Fixed point values at the DC scale for the seesaw couplings assuming initial values are above 1.5 for the data point used in figure 5.

of (a) $f_3(v_R) = 0$ and (b) $f_3(v_R) = 3.5$; the curves, in ascending order, correspond to $f_1(v_R) = 0.25, 0.5, 0.75, 1, 2.25, 3.5$. Increasing the initial value of f_3 decreases the value of f_1 at the TeV scale as can be seen by comparing figure 5(a) and figure 5(b).

Similar plots can be drawn for the other couplings: f_1, f_{c3} and f_{c1} , but their qualitative behavior follows those in figure 5. table 3 illustrates the quantitative differences in the values of the fixed points. For initial values of f_1 and f_3 greater than 1.5, these values are correct up to 2%. The higher values for the right-handed sector (f_c) are due to the slower running caused by the broken $SU(2)_R$ symmetry.

This fixed point like behavior translates into an upper bound for the slepton masses. This can be seen in figure 6, which displays the dependence of the selectron masses on the initial value of the seesaw coupling. For this plot $f_1(v_R) = f_3(v_R)$ has been assumed for simplicity. For $f \geq 0.5$ the yukawa coupling contribution is comparable in size to

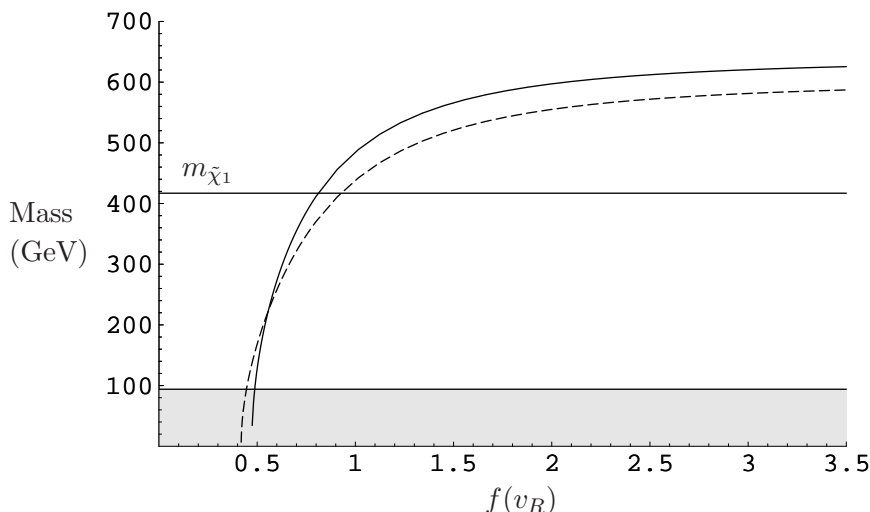


Figure 6: Plot of $m_{\tilde{e}^c}$ (dashed) and $m_{\tilde{e}}$ as a function of $f_1(v_R) = f_3(v_R)$ for $F_\phi = 33$ TeV. The greyed-out region has been excluded by LEP II and the line at around 417 GeV is the mass of the neutralino, the LSP in this case.

the gauge coupling contribution in the AMSB mass expression, e.g. eq. (2.19). The mass' quartic dependence on the seesaw couplings is reflected in its steep rise near 0.5 and its rapid surpassing of the LEP II bound. At a value of $f \sim 1$ this steep ascent slows down indicating the onset of the fixed point behavior, beyond which the low energy observable $f(F_\phi)$ values are approximately 0.6.

The masses of the other sleptons follow the behavior of figure 6, a general feature of which is the mild degeneracy between the left and right -handed slepton masses. This seems a bit contrary to eqs. (2.22) and (2.23), which show that the factor for f_1^4 term for the left-handed sleptons is twice as large as that of the right-handed sleptons. However, this term is capped by the fixed-point of f_1 and the negative $SU(2)_L$ contribution happens to be a little less than half of this value (an accidental cancelation) yielding the degeneracy.

This is an interesting situation phenomenologically since it numerically falls in between mSUGRA/mGMSB and Minimal AMSB. In mSUGRA, left-handed slepton masses get larger positive contributions from M_2 as they run from the ultraviolet. In mGMSB boundary conditions dictate that the left-handed to right-handed mass ratio is about 2 : 1. Meanwhile, in Minimal AMSB, both sectors get the same contribution from m_0 , the universal masses needed to make the sleptons non-tachyonic, which drops out in the mass splitting at tree level. Furthermore, there are accidental cancellations in the anomaly induced splittings related to the gauge contributions and in the D -term contributions [7, 35]. The upshot of this is that the mass splitting is usually dominated by loop-level effects and is quite small [7].

As a concrete example for Minimal AMSB, including the first loop leading log, the difference between the masses squared with $\tan\beta = 3.25$ and $F_\phi = 33$ TeV is given by $\Delta_e = m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2 \sim 751$ GeV² [7, 35]. The corresponding percent difference, defined as $\frac{\Delta_e}{(m_{\tilde{e}_L} + m_{\tilde{e}_R})^2}$, is then highly dependent on the masses of the selectrons. For selectron

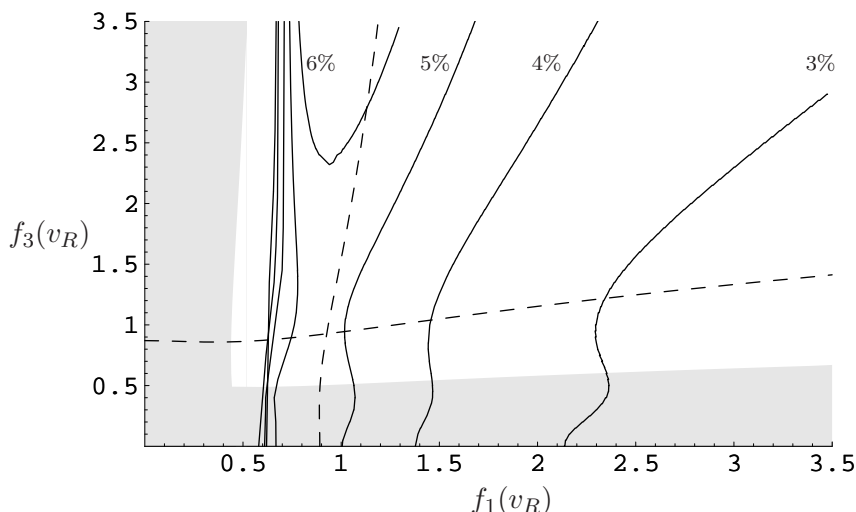


Figure 7: Constant contours of $\frac{m_{\tilde{e}} - m_{\tilde{e}^c}}{m_{\tilde{e}} + m_{\tilde{e}^c}} \times 100\%$ in the $f_3(v_R) - f_1(v_R)$ plane. The unlabeled contours on the left side of the plot, from left to right, correspond to 2%, 3%, 4% and 5%. The dashed vertical (horizontal) contour corresponds to a $\tilde{\tau}_1$ (\tilde{e}^c) constant contour of mass equal to that of the LSP (417 GeV). The shaded region is excluded by LEP II bounds of 81.9 GeV (94 GeV) on the mass of $\tilde{\tau}_1$ (\tilde{e}^c).

masses above the mass of the LSP given in figure 6, ~ 450 GeV, the percent difference is less than 1%. However, in LR-AMSB, the percent difference can rise as high as 5% as demonstrated in figure 7, which gives contours for constant mass percent differences. Resolution of slepton masses from end-point lepton distribution of the selectron decays at lepton collider is roughly 2% [36] making the measurement of such mass differences feasible. Therefore, measurements of mild mass differences of about 3–5% will single this model out from the large mass differences of mSUGRA and mGMSB while potentially discriminating it from the small mass differences of Minimal AMSB (although this will highly dependent on the values of the seesaw couplings).

Constant mass contours for the right-handed selectron are plotted in figure 8 in the $f_3(v_R) - f_1(v_R)$ plane. This plot allows a study of how the masses change with respect to both seesaw couplings. The horizontal and vertical grayed-out contours are ruled out due to LEP II bounds on the lightest stau and selectron masses of 81.9 GeV and 94 GeV respectively. Mass contours increase from left to right and correspond to $m_{\tilde{e}} = 200, 300, 417$ (the mass of the lightest neutralino, indicated with a dashed contour) 500, 550, 600, 610, 615, 620, 625, 630 GeV. The horizontal dashed curve represents a constant $m_{\tilde{\tau}_1}$ contour at the mass of the lightest neutralino. Since the selectron is a first generation slepton its mass is mainly governed by f_1 , eq. (2.20), explaining the small dependence on f_3 for smaller values of f_1 . Two things are clear from this plot: the fixed point like behavior — reflected in the fact that for large f_1 an equal change in mass requires a larger change in f_1 —and the decrease of the f_1 fixed point with the increase of f_3 . This latter point is responsible for the curving to the right of the contours at high f_1 values and was mentioned earlier with regards to figure 5.

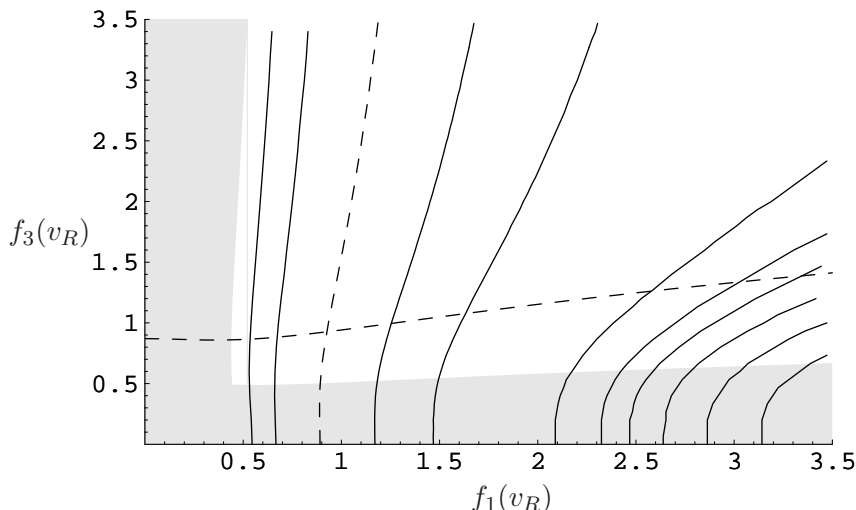


Figure 8: Mass contours for the right-handed selectron mass, $m_{\tilde{e}}$ in the $f_3(v_R)$ - $f_1(v_R)$ plane. The horizontal and vertical grayed-out contours are ruled out due to LEP II bounds on the lightest stau and selectron masses of 81.9 GeV and 94 GeV respectively. Constant mass contours for the selectron mass $m_{\tilde{e}} = 94$ (the LEP lower bound), 200, 300, 417 (the mass of the lightest neutralino, indicated with a dashed contour) 500, 550, 600, 610, 615, 620, 625, 630 GeV, for $F_\phi = 33$ TeV. The dashed horizontal curve corresponds to a $m_{\tilde{\tau}_1}$ constant contour equal to the mass of the lightest neutralino. The influence of $f_3(v_R)$ is apparent at large values of $f_1(v_R)$ and $f_3(v_R)$. Larger increases in $f_1(v_R)$ are needed for as the mass increases because of the fixed point like behavior of f_1 .

As a final remark on the sleptons, notice that the contours in figure 8 corresponding to the LSP suggest more stringent lower bounds on the seesaw couplings than the LEP II bounds. These are necessary so that the lightest neutralino will be the LSP and therefore a possible dark matter candidate. The values indicated in the plot correspond to low energy values of the seesaw couplings that are only about 10% off from their fixed-point value, $f_{c1}, f_{c3}, f_1, f_3 \sim 0.6$. Therefore, for successful dark matter, the seesaw couplings can be expected to be larger than about 0.5. This can be checked by a quick calculation, since the lightest neutralino mass is approximately the wino mass (see section 5.4) and depends only on F_ϕ . Meanwhile, the selectron mass depends on $f_1 \sim f_3 \equiv f$, which we can set equal to each other as an approximation, and F_ϕ . Given that the selectrons must be heavier than the LSP, for a viable dark matter candidate, yields

$$f(F_\phi) \gtrsim 0.58. \tag{5.1}$$

5.3 Squarks

Squark masses in Minimal AMSB decrease with energy due to the increase of $SU(2)_L$ and $U(1)_Y$ gauge couplings, which contribute negatively [37] to their masses. At a certain energy scale, the negative contributions take over and the AMSB expressions for the squark soft masses become negative. In our case this happens at an earlier scale due to the increase size of the $SU(2)_L$ and $U(1)_Y$ beta functions from the extra triplet and the doubly charged fields. Normally we would have expected this to show up at high temperatures

and lead to breakdown of color gauge symmetry. However, at high temperatures the vacuum of the theory is also affected by temperature corrections. Consequently the mass-square term of the squarks will have the form $\mu^2(T)_{\tilde{q}} \simeq (-M_{\text{AMSB}}^2 + \lambda T^2)$. The first term only grows logarithmically with temperature whereas the second term grows quadratically. The coefficient λ is positive so that the net effect is that $\mu^2(T)_{\tilde{q}}$ remains positive at high temperature and leaving color gauge symmetry intact in the early universe.

It is also worth noting that because non-asymptotically free gauge couplings contribute negatively to masses, the right-handed squarks are slightly heavier than left-handed squarks. This is different than mSUGRA and mGMSB where all gauge couplings yield positive contributions making left-handed squarks heavier, see figure 4. Furthermore, the squarks in this model can be degenerate with the sleptons.

5.4 Bosinos and the LSP

Because all superpartners eventually decay into the LSP, its makeup is an important part of SUSY collider phenomenology and dark matter prospects. Therefore understanding that makeup is an important task. Cosmological constraints rule out a charged or colored LSP [38], hence limiting the choices to the sneutrino or the lightest neutralino. The former, in typical models, makes a poor dark matter candidate (relic abundances are too light; much of its mass range ruled out by direct detection [39, 40]). It is therefore more interesting to consider the lightest neutralino as the LSP, the candidate in common SUSY scenarios (except in mGMSB where it is the next to lightest SUSY particle but has the same collider significance [41]).

The lightest neutralino will be some mixture of the wino, bino and Higgsino. Its gaugino composition follows from the gaugino mass ratio which is easily calculated and relatively independent of the point in parameter space. In AMSB this ratio depends on both the gauge couplings and the gauge coupling beta functions, b . The latter is important since this is where the effects of the light triplets and doubly-charged Higgs are felt (see table 4 for b for values in LR-AMSB compare to AMSB based on MSSM particle content). It is calculated to be: $M_3 : M_2 : M_1 \sim 1.3 : 1 : 1.3$. The striking characteristic of this ratio is its degeneracy when compared to mSUGRA/mGMSB, $M_3 : M_2 : M_1 \sim 3 : 1 : 0.3$ or even Minimal AMSB $M_3 : M_2 : M_1 \sim 8 : 1 : 3.5$.

Specifically then, the LSP will have a large wino component where in Minimal AMSB it is all wino, and there will also be some non-negligible mixing with the bino. Note that in mSUGRA (mGMSB) the sole contribution to this ratio is from the gauge couplings and therefore the LSP (NLSP) is always mostly bino. The Higgsino contribution is not independent of other parameters and therefore is not as predictable, but numerical results show that it is typically a little bit lighter than the wino (its value decreases compared to the wino as F_ϕ is increased). Therefore, the LSP will be some combination mostly Higgsino with significant wino content and a little bit of bino. The mixed Higgsino state will correspond to χ_2^0 ; χ_3^0 and χ_2^+ will be mostly wino with some Higgsino (percent values will be complementary to those of χ_1), and χ_4^0 will be mostly bino.

An immediate consequence of the degeneracy of the gauginos is a more natural heavy LSP, closer in mass to both the gluinos and squarks. Naturalness suggests that squark

	b_1	b_2	b_3
MSSM	$\frac{33}{5}$	1	-3
LR-AMSB	$\frac{78}{5}$	6	-3

Table 4: Values for the b parameter in the MSSM and LR-AMSB. Note the larger values in LR-AMSB for $SU(2)_L$ and $U(1)_Y$.

masses are not much larger than 1 TeV, to minimize fine tuning in the Higgs mass, therefore:

$$F_\phi \lesssim 63 \text{ TeV}$$

yielding:

$$\begin{aligned} M_1 &\lesssim 1350 \text{ GeV} \\ M_2 &\lesssim 980 \text{ GeV} \end{aligned} \tag{5.2}$$

This is a much larger value than the upper bound in Minimal AMSB $M_2 \lesssim 200 \text{ GeV}$ [35] and therefore has less of its parameter space ruled out by experimental data.

Another point to consider is that the Higgsino and wino form isospin doublets and triplets with the appropriate charginos. Therefore when they play the role of the lightest neutralino, there is potential for a very small mass difference between the lightest neutralino and the lightest chargino. This is very pronounced in Minimal AMSB where the mass difference of the mostly wino neutralino and chargino is on the order of 100s of MeVs including leading radiative corrections. Analytical approximations for this quantity for large μ have been given in [7, 42, 35]. Such approximations are not as useful in LR-AMSB since the relevant mass scales: μ , M_1 , and M_2 are relatively of the same order (the singlino contribution is much larger than these); however, an analytic expression for the minimum of the mass difference is attainable.

First note that a Higgsino mixing exists in the neutralino matrix, absent in the chargino sector. This mixing goes to zero as $\tan \beta \rightarrow 1$ hence indicating that for $\tan \beta = 1$ the mass difference is minimal (when $\tan \beta \rightarrow 1$ and $\tan \theta_W \rightarrow 0$ the global custodial $SU(2)$ becomes an exact symmetry making the mass difference zero). The eigenvalues of the two matrices can then be expanded for $\tan \beta = 1$ using the approximation $M_1 \sim M_2 > \mu \gg M_Z$, this yields, to first order:

$$\Delta_{\tilde{\chi}_1} \equiv m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 2 \sin^2 \theta_W \frac{M_Z^2}{M_1} \tag{5.3}$$

The second order term is positive definite so that $\Delta_{\tilde{\chi}_1}$ can in fact be used as a minimal value for the mass splitting. Notice that the $\Delta_{\tilde{\chi}_1} \rightarrow 0$ as $\tan \theta_W \rightarrow 0$ as argued above (and that $\Delta_{\tilde{\chi}_1} \rightarrow 0$ as $M_1 \rightarrow \infty$ since this also restores the custodial symmetry when $\tan \beta = 1$).

The form of eq. (5.3) is convenient since the only free parameter it depends on is F_ϕ (through M_1), which also controls the squark masses. Applying the natural upper bound for M_1 from eq. (5.2) yields:

$$\Delta_{\tilde{\chi}_1} > 1.4 \text{ GeV} \tag{5.4}$$

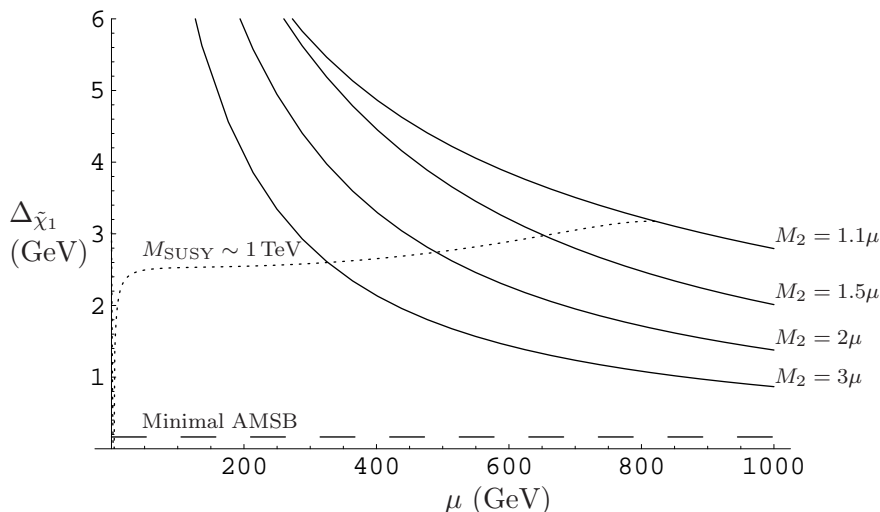


Figure 9: Mass difference of the lightest chargino and neutralino as a function of μ for $\lambda = 0.26, \tan\beta = 3.25$ and the singlino mass term $2\left(\mu_N + \frac{1}{\sqrt{2}}n\kappa\right) = 2M_1$. From top to bottom, $M_2 = 1.1\mu, 1.5\mu, 2\mu$ and 3μ . The line at 0.165 GeV is the asymptotic value for large M_2 in Minimal AMSB, while the dotted curve is represents where squark masses are at about a TeV. Below this curve, the Higgs mass is somewhat fine-tuned.

This is larger than the Minimal AMSB value of a few 100s of MeV. Exact values for the mass difference are given in figure 9 as a function of $\mu \equiv \frac{1}{\sqrt{2}}\lambda n$ with: $\lambda = 0.26, \tan\beta = 3.25$ and the singlino mass term $2\left(\mu_N + \frac{1}{\sqrt{2}}n\kappa\right) = 2M_1$. The line at 165 GeV represents the asymptotic value for large M_2 in Minimal AMSB at the one loop level [7] and below the dotted line the squark masses are above a TeV and hence the Higgs mass becomes fined tuned.

5.5 Collider signatures

The small size of $\Delta_{\tilde{\chi}_1}$ can potentially be problematic at a collider because the soft decay products, X , in the process $\tilde{\chi}_1^+ \rightarrow X\tilde{\chi}_1^0$, will not be visible. This is a feature shared by both LR-AMSB and Minimal AMSB. The difference is that the larger value of $\Delta_{\tilde{\chi}_1}$ for LR-AMSB might produce prospects of detection if $X = \tau$ or a hard μ ; however, this advantage is counterbalanced by a faster chargino decay eliminating chances of long-lived charged tracks with no muon chamber activity. Regardless, similar situations have been analyzed and found to be manageable for both lepton colliders [43] and the Tevatron [42, 44].

On the other hand, LHC studies of Minimal AMSB have focused on mSUGRA like signals [45, 37]. Such signals are heavily dependent on lepton final states and are based on left-handed squark decays to mostly wino charginos and neutralinos. These in turn can decay leptonically producing tripleton signals or same sign dilepton signals [46, 1], both of which have potentially manageable backgrounds. For in Minimal AMSB, though, the wino states are the lightest and will not decay leptonically. Hence the the right-handed squarks take the place of the left-handed ones decaying into the mostly bino neutralino

	mSUGRA and mGMSB	MAMSB	LR-AMSB
$M_3 : M_2 : M_1$	3 : 1 : 0.3	8 : 1 : 3.5	1.3 : 1 : 1.3
$ M_1 , M_2 $ (GeV) Naturalness upperbound	130, 260*	640, 200	1350, 980
Same generation slepton mass percent difference	$\sim 150\%$	$\sim 2\%$	$\sim 4\%$
Possibility of slepton-squark degeneracy	No	No	Yes

Table 5: A list of phenomenological characteristics of interest in mSUGRA, mGMSB, Minimal AMSB and LR-AMSB.

* mGMSB only, in mSUGRA sfermion and gaugino mass are determined by two separate parameters.

(which can decay leptonically). Yet, since there is no corresponding chargino to the bino, signals such as the trilepton and the same sign dilepton signal may not be possible.

The situation in LR-AMSB is more analogous to mSUGRA: right-handed squarks will decay to the LSP which has some bino content. Meanwhile, the left-handed squarks may decay either to the lightest chargino/neutralino, or, more likely (because of their higher wino content), to χ_3^0 and χ_2^+ . These may then decay leptonically depending on the slepton masses (*e.g.* $f(v_R) = 1.4$ in table 2) giving the familiar signals: trilepton and same sign dilepton. Note that it is also possible that decay of χ_1^+ will produce leptonic signals since $\Delta_{\tilde{\chi}_1}$ is larger. These considerations would help differentiate this model from Minimal AMSB, while the degeneracies in the gaugino sector and same generation slepton will differentiate it from mSUGRA and mGMSB. These differences between the various scenarios are summarized in table 5.

5.6 Triplets and doubly-charged higgses

The interplay between AMSB and the left-handed and doubly-charged Higgses leads to interesting phenomenology and is worth summarizing here. Because they play the central role of saving the slepton masses from a tachyonic fate, their masses must be around the F_ϕ scale. This puts a bound on the right-handed scale, $v_R \lesssim 10^{12}$ GeV, which is not the case when these particles appear in mSUGRA and mGMSB [20, 47]. It is also possible, through mixing due to bilinear b -terms eq. (2.17), that one triplet and one doubly charged Higgs will be light, $\mathcal{O}(1$ TeV) and therefore accessible at the LHC. Their presence would also be felt indirectly in upcoming muonium-antimuonium oscillation experiments since their couplings to first and second generation leptons must be large. For sleptons above the LEP II bound:

$$f_1(F_\phi) \sim f_2(F_\phi) \sim f_{c1}(F_\phi) \sim f_{c2}(F_\phi) \sim 0.5 \quad (5.5)$$

and for sleptons above the lightest neutralino (for a good dark matter candidate):

$$f_1(F_\phi) \sim f_2(F_\phi) \sim f_{c1}(F_\phi) \sim f_{c2}(F_\phi) \sim 0.6 \quad (5.6)$$

Based on figure 5(a) and figure 8. On the other hand, all the triplet and doubly-charged Higgsinos will remain heavy, $\mathcal{O}(F_\phi)$, and undetectable at the LHC or low energy experiments.

5.7 Dark matter

As noted in the previous section, the LSP in our model is a predominately Higgsino wino mix with very little bino (about 1%). Since the annihilation rate for such an LSP is large, its relic density from conventional annihilation arguments is not enough to explain the observed Ω_m of the universe of 20%. This issue has been discussed earlier in ref. [48], according to which the decay of the gravitino in the late stage of the universe to non-thermal winos will generate enough density to make it a viable dark matter. A similar mechanism would work in this mostly Higgsino case since the crucial ingredients are similar: the LSP mass (this is similar), its interactions with the gravitino (again, this is similar between the two cases because of the similar masses) and its annihilation rate (these are also the same since wino and Higgsino annihilation takes place through a t-channel chargino exchange proportional with α_2 strength). Like [48], we have scanned over the parameters and found that such dark matter does evade current bounds on direct detection set by CDMS Soudan and EDELWEISS but will be detectable by future experiments.

6. Beyond v_R

In this section, we comment on the ultraviolet behaviour of the theory. As we see from figure 10 below, despite the new contributions to $SU(2)_L$ and $U(1)_Y$ beta functions, all couplings remain perturbative until about 10^{11} – 10^{12} GeV. Our effective field theory approach below this scale should hold without any problem. Once we are above this scale, the couplings could maintain perturbativity if there are extra dimensions [49] due to negative contributions from vector gauge KK modes of the theory if the inverse radius of the extra dimensions are around 10^{11} GeV or so. Such extra dimensions could also be the origin of the Planck suppressed operators that we have used in our discussion.

7. Conclusion

In summary, we have elaborated on our suggestion that minimally extending MSSM to account for neutrino masses in a way that R -parity remains an automatic symmetry of the theory allows for a solution to the tachyonic slepton problem of anomaly mediated supersymmetry breaking. Interestingly, the solution requires that parity symmetry remain exact above the v_R scale. Among the new results, we show how to obtain radiative electroweak symmetry breaking and a reasonable $B\mu$ term in this class of models. We also discuss the sparticle spectrum of the model in detail and show how it differs from that of Minimal AMSB as well as other widely discussed supersymmetry scenarios. A new feature of this model is the presence of new TeV scale $SU(2)_L$ triplets and doubly charged $SU(2)_L$ singlet fields, whose phenomenology has been the subject of many papers [50–55]. We believe that the model discussed here is a serious alternative to the Minimal AMSB whose further phenomenological implications need to be explored in detail.

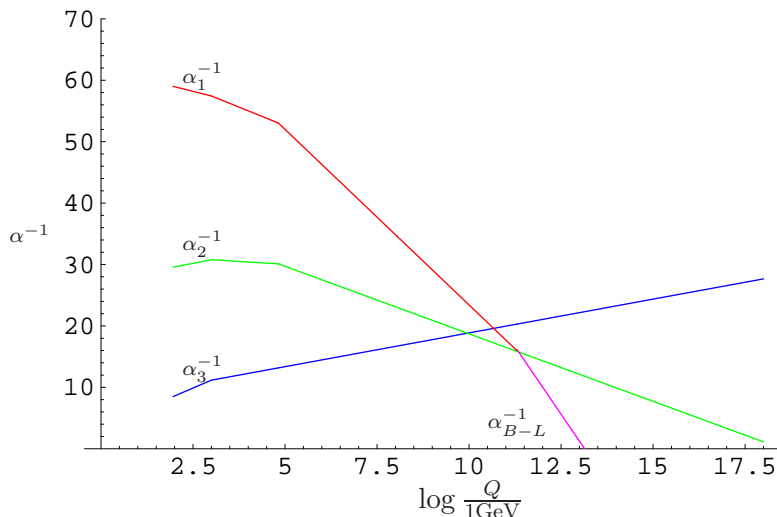


Figure 10: Inverse gauge couplings as a function of the log of the energy scale. The v_R scale is at about 10^{12} GeV at which point α_R^{-1} begins to run with its curve being indistinguishable from α_2^{-1} due to parity.

Acknowledgments

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A. Notation conventions

In this appendix we summarize our notational conventions. Given a superpotential defined as

$$W = L^i \Phi_i + \frac{1}{2!} \mu^{ij} \Phi_i \Phi_j + \frac{1}{3!} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{4!} \frac{\lambda^{ijkl}}{M} \Phi_i \Phi_j \Phi_k \Phi_l + \dots \quad (\text{A.1})$$

with a corresponding lagrangian of

$$\mathcal{L} = \int d^4\theta (Z_j^i \Phi_i \Phi^{j*} + \dots) + \left[\int d^2\theta (W + \mathcal{W}^\alpha \mathcal{W}_\alpha) + \text{h.c.} \right] \quad (\text{A.2})$$

the anomalous dimensions, γ_j^i , and β -functions, β_L^i , β_μ^{ij} , β_Y^{ijk} , at a given energy scale Q are defined by

$$\gamma_j^i = \frac{d \ln Z_j^i}{d \ln Q} = 4C_a(\Phi_i) g_a^2 \delta_j^i - Y_{j p q} Y^{i p q} \quad (\text{A.3})$$

$$\beta_L^i = \frac{d L^i}{d \ln Q} = -\frac{1}{2} L^j \gamma_j^i \quad (\text{A.4})$$

$$\beta_\mu^{ij} = \frac{d \mu^{ij}}{d \ln Q} = -\frac{1}{2} \mu^{ip} \gamma_p^j + (j \leftrightarrow i) \quad (\text{A.5})$$

$$\beta_Y^{ijk} = \frac{d Y^{ijk}}{d \ln Q} = -\frac{1}{2} Y^{ijp} \gamma_p^k + (k \leftrightarrow i) + (k \leftrightarrow j) \quad (\text{A.6})$$

Furthermore, we choose the sign of the soft SUSY breaking terms by specifying that

$$V_{\text{SB}} = \frac{1}{2}(m^2)_j^i \Phi_i \Phi_j^{*} + \ell^i \Phi_i + \frac{1}{2!} b^{ij} \Phi_i \Phi_j + \frac{1}{3!} a^{ijk} \Phi_i \Phi_j \Phi_k + \text{h.c.} \quad (\text{A.7})$$

B. Between scales: v_R To F_ϕ

The superpotential between the v_R and F_ϕ scale is:

$$\begin{aligned} W_{\text{NMSSM}++} = & \mathbf{i}y_t^a Q^T \tau_2 H_{ua} t^c + \mathbf{i}y_b^a Q^T \tau_2 H_{da} b^c + \mathbf{i}y_\tau^a Q^T \tau_2 H_{da} t^c \\ & + f_{ci} e_i^c \Delta^{c--} e_i^c + \mathbf{i}f_i L_i^T \tau_2 \Delta L_i \\ & + \mathbf{i}\lambda^{ab} N H_{ua} \tau_2 H_{db} + \frac{1}{2} \mu_N N^2 + \frac{1}{3} \kappa N^3 \end{aligned} \quad (\text{B.1})$$

Where $a = 1, 2$, the MSSM Yukawa matrices have been approximated by the third generation diagonal term, the seesaw Yukawa couplings are diagonal and the subscript i represents lepton generation and is summed. The gamma functions for the theory between the v_R and F_ϕ are:

$$\gamma_{Q_3} = -\frac{1}{8\pi^2} \left(y_t^{a*} y_t^a + y_b^{a*} y_b^a - \frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{1}{30} g_1^2 \right) \quad (\text{B.2})$$

$$\gamma_{Q_1} = -\frac{1}{8\pi^2} \left(-\frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{1}{30} g_1^2 \right) \quad (\text{B.3})$$

$$\gamma_{t^c} = -\frac{1}{8\pi^2} \left(2y_t^{a*} y_t^a - \frac{8}{3} g_3^2 - \frac{8}{15} g_1^2 \right) \quad (\text{B.4})$$

$$\gamma_{u^c} = -\frac{1}{8\pi^2} \left(-\frac{8}{3} g_3^2 - \frac{8}{15} g_1^2 \right) \quad (\text{B.5})$$

$$\gamma_{b^c} = -\frac{1}{8\pi^2} \left(2y_b^{a*} y_b^a - \frac{8}{3} g_3^2 - \frac{2}{15} g_1^2 \right) \quad (\text{B.6})$$

$$\gamma_{d^c} = -\frac{1}{8\pi^2} \left(-\frac{8}{3} g_3^2 - \frac{2}{15} g_1^2 \right) \quad (\text{B.7})$$

$$\gamma_{L_3} = -\frac{1}{8\pi^2} \left(y_\tau^{a*} y_\tau^a + 6|f_3|^2 - \frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 \right) \quad (\text{B.8})$$

$$\gamma_{L_1} = -\frac{1}{8\pi^2} \left(6|f_1|^2 - \frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 \right) \quad (\text{B.9})$$

$$\gamma_{\tau^c} = -\frac{1}{8\pi^2} \left(2y_\tau^{a*} y_\tau^a + 4|f_{c3}|^2 - \frac{6}{5} g_1^2 \right) \quad (\text{B.10})$$

$$\gamma_{e^c} = -\frac{1}{8\pi^2} \left(4|f_{c1}|^2 - \frac{6}{5} g_1^2 \right) \quad (\text{B.11})$$

$$\gamma_N = -\frac{1}{8\pi^2} \left(2|\kappa|^2 + 2\lambda^{ab*} \lambda^{ab} \right) \quad (\text{B.12})$$

$$\gamma_{H_{ua}}^{H_{ub}} = -\frac{1}{8\pi^2} \left(3y_t^{a*} y_t^b + \lambda^{ac} \lambda^{bc} - \delta^{ab} \left(\frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 \right) \right) \quad (\text{B.13})$$

$$\gamma_{H_{da}}^{H_{db}} = -\frac{1}{8\pi^2} \left(3y_b^{a*} y_b^b + y_\tau^{a*} y_\tau^b + \lambda^{ca*} \lambda^{cb} - \delta^{ab} \left(\frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 \right) \right) \quad (\text{B.14})$$

$$\gamma_{\Delta} = -\frac{1}{8\pi^2} \left(2|f_3|^2 + 2|f_2|^2 + 2|f_1|^2 - 4g_2^2 - \frac{6}{5}g_1^2 \right) \quad (\text{B.15})$$

$$\gamma_{\bar{\Delta}} = -\frac{1}{8\pi^2} \left(-4g_2^2 - \frac{6}{5}g_1^2 \right) \quad (\text{B.16})$$

$$\gamma_{\Delta^{c--}} = -\frac{1}{8\pi^2} \left(2|f_{c3}|^2 + 2|f_{c2}|^2 + 2|f_{c1}|^2 - \frac{24}{5}g_1^2 \right) \quad (\text{B.17})$$

$$\gamma_{\bar{\Delta}^{c--}} = -\frac{1}{8\pi^2} \left(-\frac{24}{5}g_1^2 \right) \quad (\text{B.18})$$

These expressions were used for the slepton masses in eqs. (2.22) and (2.23). The third generation squark masses can also be written down (here we assume real yukawa couplings for simplicity):

$$m_{Q_3}^2 = \frac{1}{4}F_{\phi}^2 \left\{ -\frac{b_1\alpha_1^2}{72\pi^2} - \frac{3b_2\alpha_2^2}{8\pi^2} - \frac{2b_3\alpha_3^2}{3\pi^2} \right. \quad (\text{B.19})$$

$$+ \frac{4y_t^a}{16\pi^2} \left[\frac{y_t^a}{16\pi^2} \left(3(y_t^c)^2 + (y_b^c)^2 - \frac{13}{9}g_1^2 - 3g_2^2 - \frac{8}{3}g_3^2 \right) + \frac{y_t^c}{16\pi^2} (3y_t^a y_t^c + \lambda^{ad}\lambda^{cd}) \right]$$

$$+ \frac{4y_b^a}{16\pi^2} \left[\frac{y_b^a}{16\pi^2} \left(3(y_b^c)^2 + (y_t^c)^2 + (y_{\tau}^c)^2 - \frac{7}{9}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right) \right.$$

$$\left. \left. + \frac{y_b^c}{16\pi^2} (3y_b^a y_b^c + y_{\tau}^a y_{\tau}^c + \lambda^{da}\lambda^{dc}) \right] \right\}$$

$$m_{t^c}^2 = \frac{1}{4}F_{\phi}^2 \left\{ -\frac{2b_1\alpha_1^2}{9\pi^2} - \frac{2b_3\alpha_3^2}{3\pi^2} \right. \quad (\text{B.20})$$

$$\left. + \frac{8y_t^a}{16\pi^2} \left[\frac{y_t^a}{16\pi^2} \left(3(y_t^c)^2 + (y_b^c)^2 - \frac{13}{9}g_1^2 - 3g_2^2 - \frac{8}{3}g_3^2 \right) + \frac{y_t^c}{16\pi^2} (3y_t^a y_t^c + \lambda^{ad}\lambda^{cd}) \right] \right\}$$

$$m_{b^c}^2 = \frac{1}{4}F_{\phi}^2 \left\{ -\frac{b_1\alpha_1^2}{18\pi^2} - \frac{2b_3\alpha_3^2}{3\pi^2} \right. \quad (\text{B.21})$$

$$+ \frac{8y_b^a}{16\pi^2} \left[\frac{y_b^a}{16\pi^2} \left(3(y_b^c)^2 + (y_t^c)^2 + (y_{\tau}^c)^2 - \frac{7}{9}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right) \right.$$

$$\left. \left. + \frac{y_b^c}{16\pi^2} (3y_b^a y_b^c + y_{\tau}^a y_{\tau}^c + \lambda^{da}\lambda^{dc}) \right] \right\}$$

Were the first generation squark masses can be found by using the third generation mass expressions with yukawa couplings set to zero and $b_A = (\frac{78}{5}, 6, -3)$ for $A = (1, 2, 3)$. Typically, the largest contribution to these are given by:

$$m_{\tilde{q}}^2 \sim F_{\phi}^2 \frac{\alpha_3^2(F_{\phi})}{2\pi^2}. \quad (\text{B.22})$$

The present LEP bound on the Higgs mass of 114 GeV can then roughly be translated to give a lower bound of about 600 GeV on the top squark mass. Using $\alpha_3(F_{\phi}) \simeq 0.08$ in the above expressions, we can translate this squark mass bound to a lower limit on F_{ϕ} of about 30 TeV. We have used this in all our calculations in the text.

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